The Katz School

Workshop in Mathematics

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1. Basic Mathematical Concepts and Terminology

1.1 Functions

1.2 Often used constants

1.3 Exponents

1.4 Polynomials

1. 5 One Variable Linear Equations

Supply and Demand as a function of Price

Let the supply quantity (Q) as a function of price be

$$Q = 3P + 10$$

Let the Demand quantity (θ) as a function of price be

$$Q = -2P + 60$$

Find the equilibrium Price and Quantity using (i) equations and (ii) graphical methods.

Break Even Analysis

Total Revenue, $R(x) = 55x$
Total Cost, $C(x) = 30x + 250$

Find the breakeven point x for the firm (using (i) profit function, (ii) equating revenue to cost and (iii) graphical method).

1. 6 Quadratic Equations.

If the equation is quadratic, that is, it is of the ~~from~~ form

$$ax^2 + bx + c = 0$$

There are two possible solutions:

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

and

$$x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$d = b^2 - 4ac$, is called the discriminant.

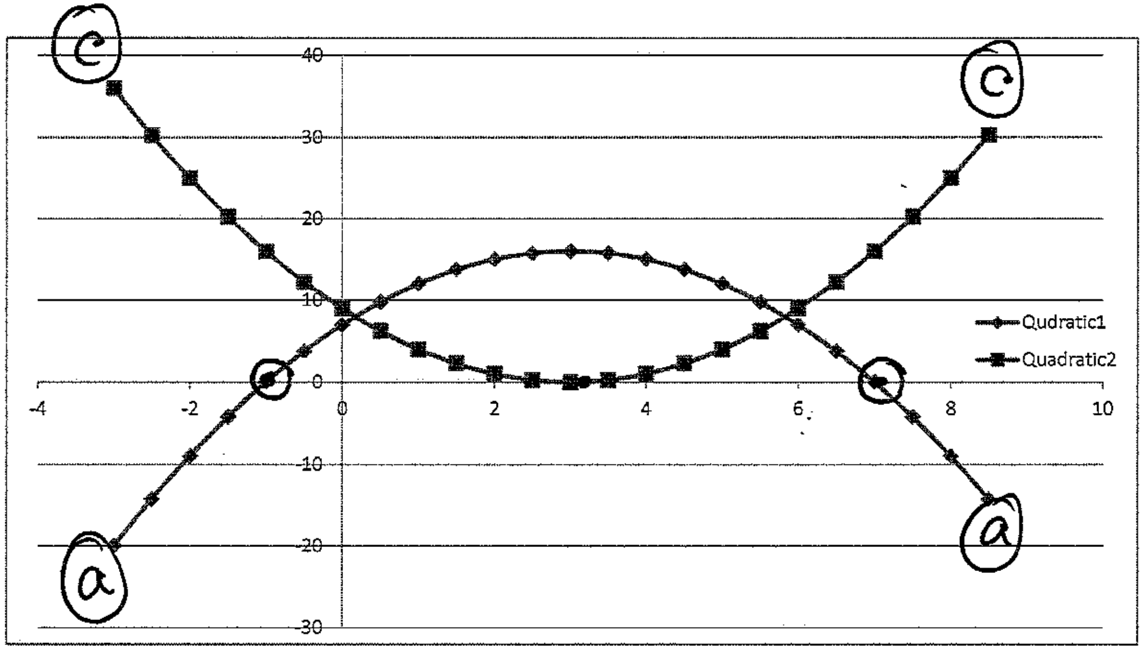
Note that if $d=0$ the two solutions are equal.

If $d < 0$, then there is no real solution to the equation.

Examples:

(a) $-x^2 + 6x + 7 = 0$
(b) $5x^2 + 47x + 18 = 0$
(c) $x^2 - 6x + 9 = 0$

x	Qudratic1	Quadratic2
-3	-20	36
-2.5	-14.25	30.25
-2	-9	25
-1.5	-4.25	20.25
-1	0	16
-0.5	3.75	12.25
0	7	9
0.5	9.75	6.25
1	12	4
1.5	13.75	2.25
2	15	1
2.5	15.75	0.25
3	16	0
3.5	15.75	0.25
4	15	1
4.5	13.75	2.25
5	12	4
5.5	9.75	6.25
6	7	9
6.5	3.75	12.25
7	0	16
7.5	-4.25	20.25
8	-9	25
8.5	-14.25	30.25



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Two or more Linear equations

There are three possible cases you may encounter in solving several linear equations in several unknowns:

Case 1 – The number of equations is the same as the number of variables.

Case 2 – there are more variables than there are equations

Case 3 – there are more equations than there are variables

Case 1 – The number of equations is the same as the number of variables

For purposes of the MBA program, it is sufficient to deal only with systems of two simultaneous linear equations in two unknowns.

Example .

A company makes two different products and uses two different resources: Raw material and labor hours. Product1 requires 20 pounds of raw material and 4 hours of labor. Product2 requires 30 pounds of raw material and 1 hour of labor.

Assume that the Company A has 480 lbs of raw material and 36 hours of labor (and he has to use all the resources). We want to know many units of Product1 and Product2 can be made using all of the resources.

Let x_1 denote the number of units of Product1 and let x_2 denote the number of Product2 produced. As a function of x_1 and x_2, the number of pounds of raw material required is

$$20x_1 + 30x_2.$$

As a function of x_1 and x_2, the number of hours of labor is

$$4x_1 + 1x_2.$$

The two equations we need to solve are:

$$20x_1 + 30x_2 = 480$$
$$4x_1 + 1x_2 = 36.$$

Graphical Representation

Case 2 – More variables than equations

When there are more variables than there are equations, there are (in general) many possible solutions to the simultaneous equations. Such systems of equations occur, for example, in linear programming. Although in some situations, it is useful to enumerate or to describe all the possible solutions, in linear programming, you are provided with additional information that permits you to determine which of the possible solutions are more desirable than the others.

Example : Same example as above except that the company produces 3 different products

Case 3 – More equations than variables

When there are more equations than variables, there is, in general, no solution that exactly solves the equations. However, Case 3 happens very often in business applications, and there is a way to find the best approximate solution, in the sense that the approximate solution determines a set of values for the variables so that there is the least amount of error across all the equations. If the error is measured by the square of the difference between the left-hand-sides and the corresponding right-hand-sides of the equations, the best approximate solution is called the least squares solution, and is the one found by (ordinary least squares) linear regression.

Example : Consider the original two product problem with a third resource, say, storage space, constraint.

Equations and Inequalities.

Multiplying by a constant

Multiplying by -1.

1.7 Distance between two points

Let the coordinated of the points A & B be

$A\ (4;5)\ ,\ B(10,12)$

What is the distance between the points A & B?

In general, the distance between any two points (x_1, y_1) and (x_2 , y_2) is given by:

$$\sqrt{(x_1 - x_2)^2 + (y_2 - y_1)^2}$$

This distance, or the length of the line segment, is also called the Eucledian distance.

2. **Slopes and Derivatives.**
(Differential Calculus)

What is the slope?

In economic and business applications, it is often important to know how a variable, y that is a function of a variable x,

$$y = f(x)$$

changes with changes in x.

The slope of a line measures how much the value of y changes for a particular change in the value of x. It is measured by the slope of a line drawn tangent to the function at that point (A tangent line touches the curve at only one point). Denoting the change in y by Δy and the change in x by Δx, the difference quotient is the ratio, Δy/Δx. More formally, for a function y=f(x) at some point x_0, the difference quotient

$$\Delta y/\Delta x = \frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$$

over some interval (x_0, $x_0+\Delta x$) gives the average rate of change of the function f(x) over that interval. If two variables, x and y, are functionally related by the straight line

$$y = a + bx$$

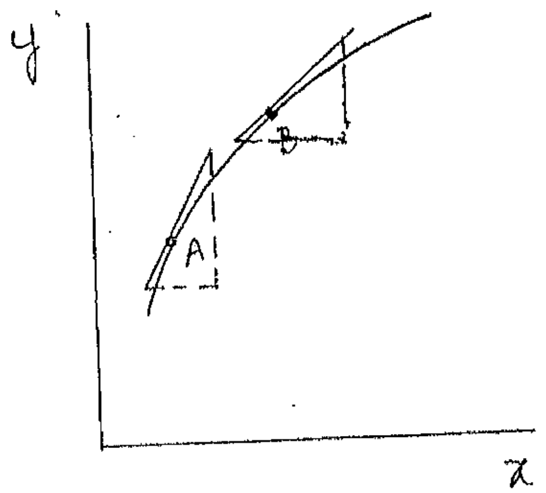
then the difference quotient is equal to the slope of y with respect to x, **b**. For a linear relationship, **b** measures the marginal change in y for a change in x for any value of x and for any amount of change Δx. That's because the slope of the relationship is a constant.

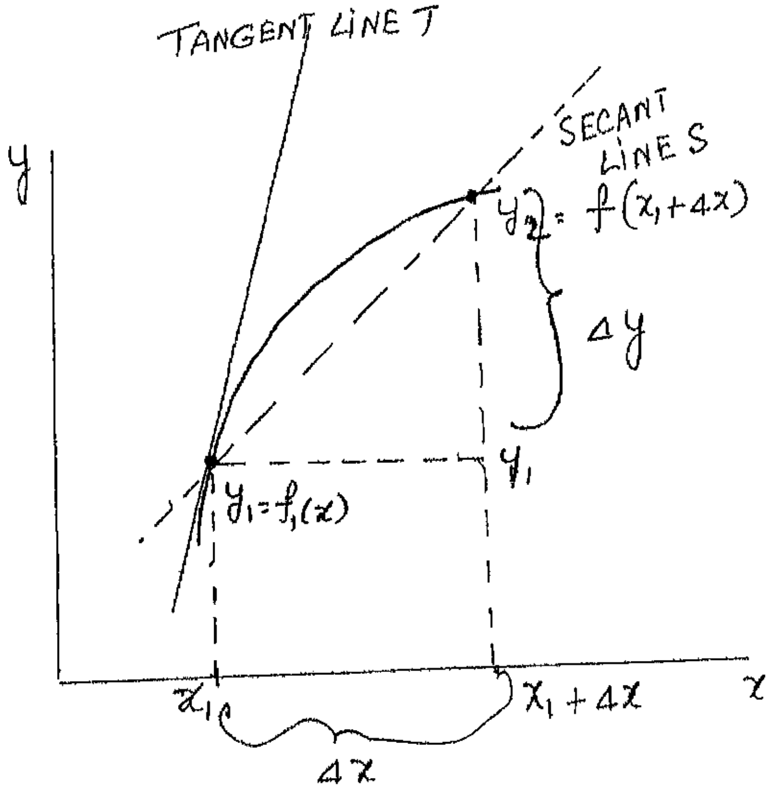
In the case where x and y are nonlinearly related, it is still meaningful to speak of a slope in the relationship between x and y, but the slope is no longer a constant because it changes with x. The more nonlinear the relationship between x and y, the more limited the neighborhood for which the slope is a valid measure of incremental change.

2.1 Slope of a curvelinear function:

The slope of a curvelinear function differs at different points on a curve. At a given point, it is measured by the slope of a line drawn tangent to the function at that point. (A tangent line touches the curve at only one point).

Geometric Procedure:





$$\text{slope of } S = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_1 + \Delta x) - f(x_1)}{\Delta x}$$

If x_2 is made to coincide with x_1 i.e $\Delta x \to 0$

$$\text{slope of } T = \lim_{\Delta x \to 0} \frac{f(x_1 + \Delta x) - f(x_1)}{\Delta x}$$

$f(x) = 2x^2$, slope $T = \lim_{\Delta x \to 0} \frac{2(x+\Delta x)^2 - 2x^2}{\Delta x}$

$$= \lim_{\Delta x \to 0} \frac{2(x^2 + 2x\,\Delta x + (\Delta x)^2) - 2x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{4x(\Delta x) + 2(\Delta x)^2}{\Delta x} = \lim_{\Delta x \to 0} (4x + 2\Delta x$$

$$= 4x$$

2.2 The Derivative (Algebraic Procedure)

Given a function $y = f(x)$ the derivative of f at x, denoted by $f'(x)$ or dy/dx is defined as

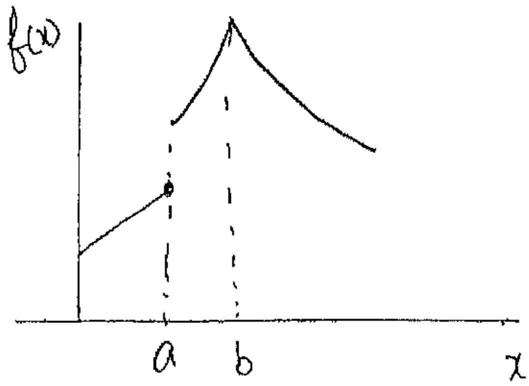
$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \text{ if the limit exists.}$$

2.3 Differentiability and Continuity

A continuous function has no breaks in its curve. It can be drawn without lifting the pencil from the paper.

A function is differentiable at a point if i) it is continuous at that point and ii) has a unique tangent at that point.

The function is not diffeentiable at a (not continuous) or at b (no unique tangent).



Some useful Derivatives and Integrals

Derivatives		Integrals	
$f(x)$	$f'(x)$	$f(x)$	$\int f(x)$
k	0	k	$kx + c$
kx	k		
x^2	$2x$	x	$\frac{x^2}{2} + c$
kx^2	$k \cdot 2x$		
x^3	$3x^2$	x^2	$\frac{x^2}{3} + c$
x^n	$n \cdot x^{n-1}$	x^N	$\frac{x^{N+1}}{N+1} + c$
kx^n	$k \cdot n \cdot x^{n-1}$		
$\ln x$	$1/x$	e^x	$e^x + c$
e^x	e^x	e^{ax}	$\frac{e^{ax}}{a} + c$
e^{ax^2}	$e^{ax^2} \cdot 2ax$	e^{x^2}	?
$e^{g(x)}$	$e^{g(x)} \cdot g'(x)$	$\ell n x$	$x \ell n x - x$

Rules of Differentiation

3.1 The Constant Function Rule

Given $f(x) = k, f'(x) = 0$

Examples: $f(x) = 2.6, \quad f'(x) = 0$

(What is the slope of a horizontal line)

3.2 The Linear Function Rule

If $f(x) = mx + c, \qquad f'(x) = m$

$f(x) = 10.5 + 2.6x \qquad f'(x) = 2.6$

(What is the slope of a straight line at different points)

3.3 Power Function Rule

$f(x) = x^N \qquad f'(x) = N \cdot x^{N-1}$

3.4 Rules for Sums and Differences

$f(x) = g(x) + h(x)$ ……………………… $f'(x) = g'(x) + h'(x)$

Example: $f(x) = x^3 + e^{x^2}$

3.5 Quotient Rule

$$f(x) = \frac{g(x)}{h(x)} \ldots\ldots f'(x) = \frac{\{h(x) \cdot g'(x) - g(x) \cdot h'(x)\}}{\{h(x)\}^2}$$

Example: $f(x) = \frac{4x}{1-2x}$

3.6 Chain Rule

Let $y = f(u)\ , u = g(x)$ ………………… $\left(\frac{dy}{du}\right) \cdot \left(\frac{du}{dx}\right)$

Example: $y = (2x^3 + 4)^2$ ……………… $U = 2x^3 + 4, \quad y = U^2$

3.7 Higher order Derivatives

The second – order derivative, (denoted by $f''(x)$), measures the rate of change (slope) of the first derivative. Note that the first-order derivative measures the slope of the original function. Thus the second-order derivative measures the slope of the "slope function". The third-order derivative $f'''(x)$ measures the slope of the second order derivative, $f''(x)$. Higher-order derivatives are obtained by applying the rules of differentiation to lower-order derivatives.

Examples: $f(x) = 5x^3 - 3x^2 + 14x + 9$

$f'(x) =$

$f''(x) =$

Differentiation Examples

1. $f(x) = x^3 + 2x^2 + 9x - 7$

2. $f(x) = e^{x^2}$

3. $f(x) = e^x \cdot x^2$

4. $f(x) = 5x^3 + e^{x^2}$

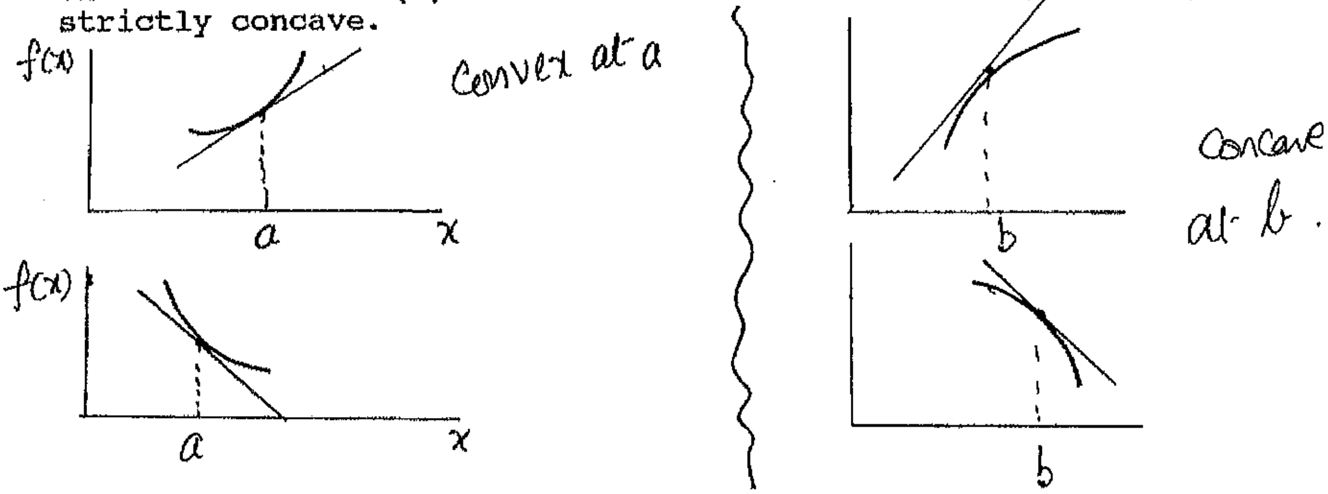
4. Uses of the Derivatives in Economics, Operations, Statistics, and Marketing

4.1 Convexity and Concavity:

A function is convex at a point x = a if in the neighborhood to {a, f(a)} the graph of the function is completely above the tangent line. If the second derivative at x = a is positive, then the function is convex at x = a .

A function is concave at x = a if in the neighborhood to {a, f(a)} the graph of the function is completely below the tangent line. If the second derivative at x = a is negative, then the function is concave at x = a .

The function f(x) is strictly convex if $f''(x) > 0$ for all x in the domain. If $f''(x) < 0$ for all x in the domain, then f(x) is strictly concave.



4.2 Maximum, Minimum and Inflection Points.

If the first derivative is zero or is undefined at a point **a**, then **a** is called a critical point. A critical point can be a relative (local) maximum, minimum or an inflection point.

Assuming $f'(a) = 0$,

i) if $f''(a) > 0$, then f(x) is at a relative minimum at x = a.

ii) if $f''(a) < 0$, then f(x) is at a relative maximum at x = a.

iii) if $f''(a) = 0$, then **a** is an inflection point.

(At an inflection point on the graph the function crosses its tangent line and changes from concave to convex or vice versa).

4.3 Finding the maximums, the minimums and the inflection points. OPTIMIZATION

One of the most commonly used calculus techniques is differentiation. Derivatives are used to find local extreme (minimums and maximums) in several business applications. We use the following two step procedure in optimization.

i) Obtain the first derivative, set it equal to zero and solve for the critical point(s). (This is known as the first order condition).

ii) Obtain the second derivative, evaluate it (the second derivative) at these critical points. If at a critical point k,

$f''(k) > 0$; relative minimum (convex)

$f''(k) < 0$; relative maximum (concave)

$f''(k) = 0$; inflection point (test inconclusive)

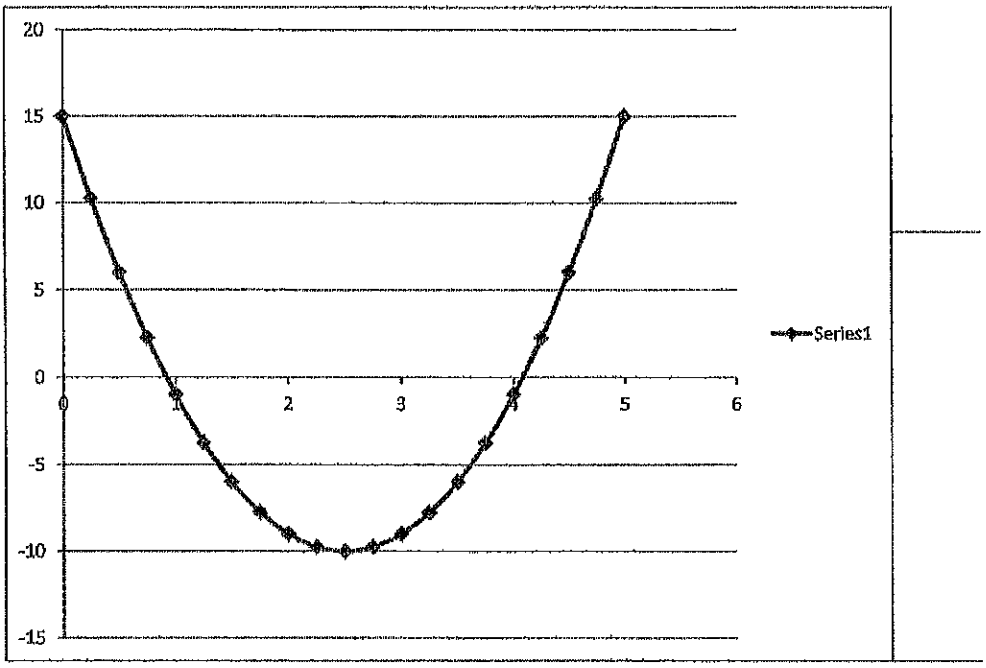
Examples:

1. $y = 4x^2 - 20x + 15$

2. $y = x^3 - 6x^2 - 135x + 4$

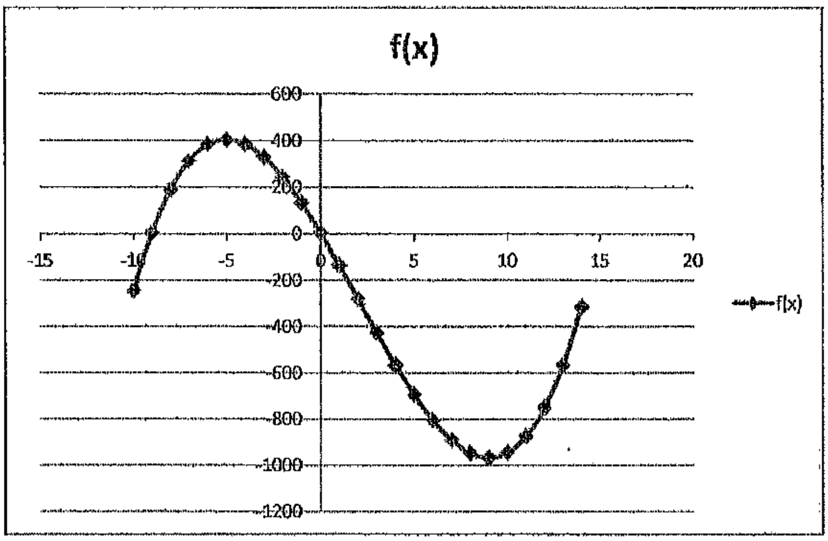
X	f(x)
0	15
0.25	10.25
0.5	6
0.75	2.25
1	-1
1.25	-3.75
1.5	-6
1.75	-7.75
2	-9
2.25	-9.75
2.5	-10
2.75	-9.75
3	-9
3.25	-7.75
3.5	-6
3.75	-3.75
4	-1
4.25	2.25
4.5	6
4.75	10.25
5	15

f(x)= 4x^2-20x+15



x	f(x)
-10	-246
-9	4
-8	188
-7	312
-6	382
-5	404
-4	384
-3	328
-2	242
-1	132
0	4
1	-136
2	-282
3	-428
4	-568
5	-696
6	-806
7	-892
8	-948
9	-968
10	-946
11	-876
12	-752
13	-568
14	-318

f(x)=x^3-6x^2-135x+15



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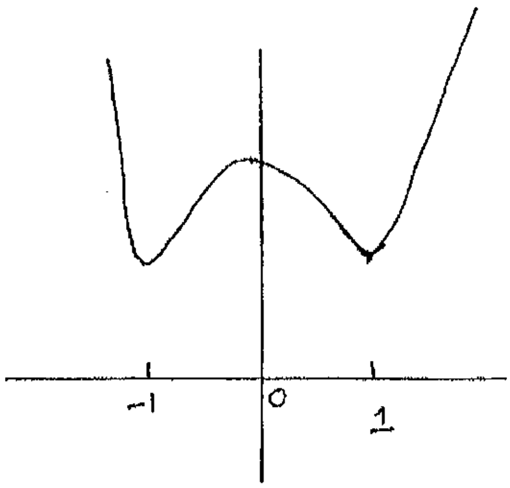
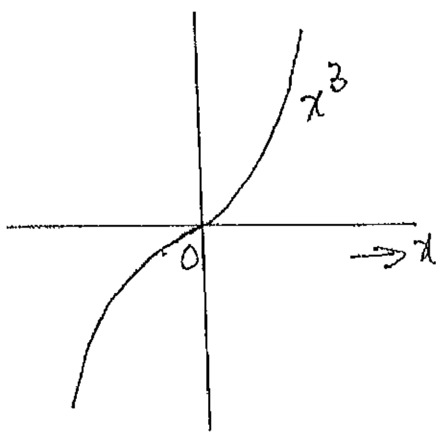
OPTIMIZATION EXAMPLES

1. Find the extreme Points of $f(x) = x^4 - 2x^2 + 2$



2. Inflection Points: $y = f(x) = x^3$



4.4 Examples from Economics: The following notation and terminology are common in Economics.

Marginal Cost (MC), Marginal Revenue (MR), Marginal Utility (MU) Total Cost (TC), Total Revenue (TR), Total Utility (TU) Price (P), The Level of Output (Q), Price Elasticity of Demand (Supply) (E), Profit (π)

Definitions

$$MC = \frac{dTC}{dQ} \quad ; \quad MR = \frac{dTR}{dQ}$$

3. Profit Maximization: Let $TR = 2000Q - 20Q^2$ and $TC = 2Q^2 + 1000Q + 5000$. If, $profit = TR - TC$, Maximize Profit.

4. $TR = 4000Q - 33Q^2; TC = 2Q^3 - 3Q^2 + 400Q + 5000$
   $\pi = TR - TC \quad Maximize\ Profit$

4.5 Price Elasticity of Demand

$$E = \frac{\frac{dQ}{Q}}{\frac{dP}{P}} = \frac{Percentage\ change\ in\ Quantity\ Demanded}{Percentage\ change\ in\ price}$$

If $P = f(Q)$, then $\frac{dQ}{dP} = \frac{1}{\frac{dp}{dQ}}$

$$\therefore E = \frac{1}{\frac{dp}{dQ}} \cdot \frac{P}{Q}$$

Example: If the demand function $Q = 5000 - 2P^2$,

Find the price elasticity of demand at P = 25

Examples from Operations Management:

Inventory Control: The Economic Order Quantity (Q) minimizes the total cost (TC) which is the sum of the Annual Ordering Costs (AOC) and the Annual Holding Costs (AHC). How do we determine Q?

Derivatives of functions with two or more x variables (Partial Differentiation)

When y is a nonlinear function of two or more x variables, then it is possible to take derivatives of each variable separately, treating the remaining x variables as if they were constants. Derivatives found in such a fashion are called *partial derivatives*, and measure the rate of change in y with a single x, provided all the other x variables remain constant.

For example, if y is related to x_1 and x_2, then the partial derivative of y with respect to x_1 tells you how y changes when x_1 changes, provided that x_2 is unchanged. Consider a specific type of such function that appears in most statistics texts, where y is a function of two variables, x_1, and x_2, and that they are related in a linear way with an interaction term. That is, there are numbers a, b_1, b_2, and b_3 such that

$$y = a + b_1x_1 + b_2x_2 + b_3x_1x_2.$$

That function is nonlinear because the interaction term, $b_3x_1x_2$, is nonlinear.

In order to find the partial derivative of y with respect to x_1, you treat x_2 as if it were a constant. The partial derivative of y with respect to x_1, written as $\partial y/\partial x_1$, is thus the partial derivative of a (0, because a is a constant) plus the partial derivative of b_1x_1 (which is b_1) plus the partial derivative of b_2x_2 (which is 0, because we are treating x_2 as a constant and the derivative of a constant is 0) plus the derivative of $b_3x_1x_2$ (which is b_3x_2, again because we treat x_2 as a constant), for a total of

$$\partial y/\partial x_1 = b_1 + b_3x_2.$$

By similar reasoning, the partial derivative of y with respect to x_2, denoted $\partial y/\partial x_2$ is calculated by treating x_1 as a constant, resulting in

$$\partial y/\partial x_2 = b_2 + b_3x_1.$$

Those partial derivatives imply that the change in y due to a change in x_1 depends on the value of x_2, and the change in y due to a change in x_2 depends on the value of x_1.

Example:

The career services office of a graduate school of business analyzed data for

y, MBA graduates' starting salaries, in thousands
x_1, the number of years of relevant work experience prior to entering the MBA program, and
x_2, the graduating class rank in percentile, between 0 and 100

and found the following relationship

$$y = 60 + 6x_1 + 0.5x_2 - 0.02x_1x_2.$$

What is the impact an additional year's experience (x_1) on starting salary (y) for students graduating in the 50^{th} and 90^{th} percentiles of the MBA class?

From page 25,

$$\partial y/\partial x_1 = b_1 + b_3x_2 = 6 - 0.02x_2.$$

For a student graduating in the 50^{th} percentile of the class, for whom x_2=50, 6 – 0.02(50) = 6 – 1 = 5, so an additional year's experience is worth an extra $5,000 in salary.

For a student graduating in the 90^{th} percentile of the class, for whom x_2=90, 6 – 0.02(90) = 6 – 1.8 = 4.2, so an extra year's experience is worth an extra $4200 in salary.

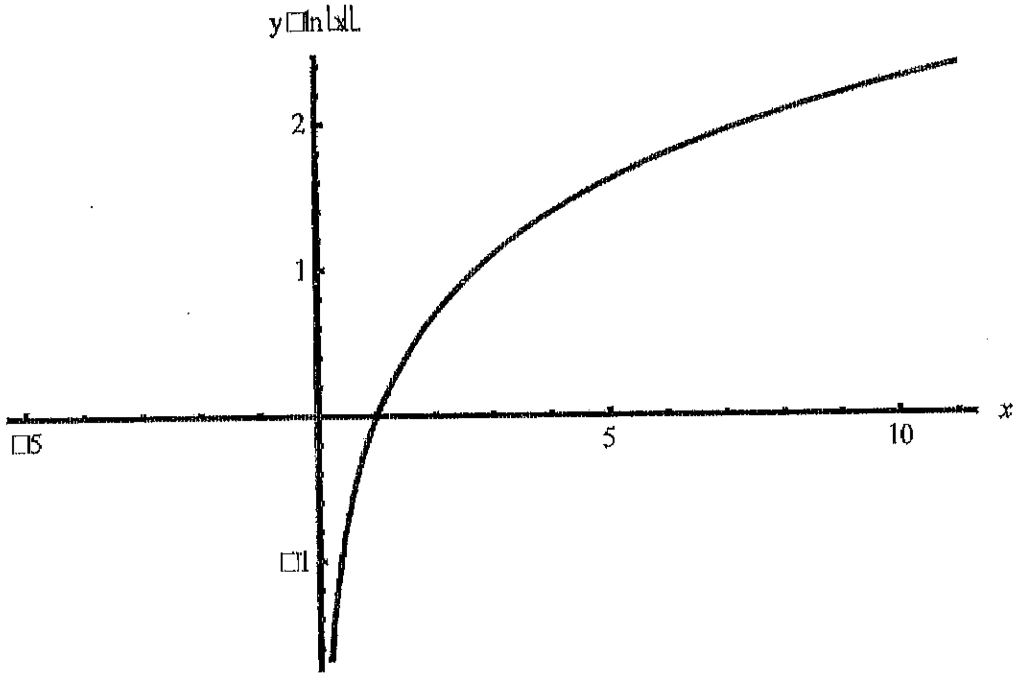
More work experience always leads to a higher salary, and so does a higher class rank - it is the marginal impact of experience that is lower for students graduating higher up in the class. The nonlinearity of the function is the reason why the marginal impact of one variable, experience, is a function of another variable, class rank.

Logarithmic functions

A logarithmic function is the logarithm (usually the natural logarithm) of an expression involving one or more x variables and it is a non linear function. Most of the time we talk about the natural logarithm (to the base e) or logarithm to the base 10.

In EXCEL the two corresponding functions are LN and LOG (or LOG10), respectively.

A logarithmic function increases steadily, but at a decreasing rate. As an example, Figure below shows the plot of the function y = ln(x).



LN function

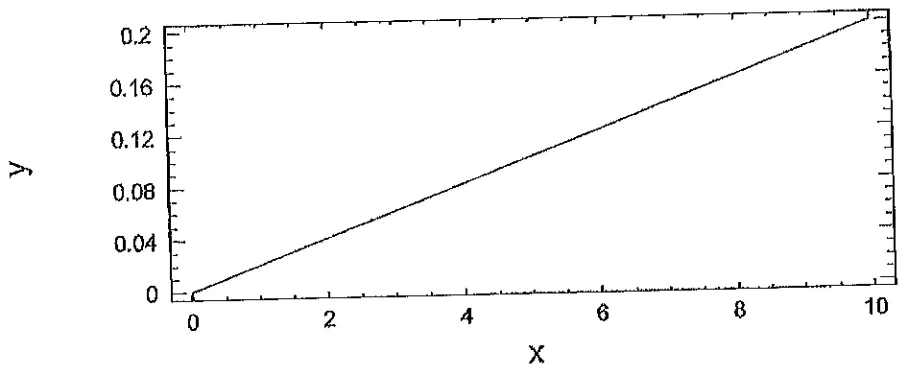
3. AREAS UNDER CURVES (Integral Calculus)

Areas under curves that are never below the x-axis, that is, areas under functions that only take on nonnegative values, are important for understanding a wide variety of economic concepts and also for determining probabilities. The area under such a function is also a function, because it depends on the part of the curve you consider, and it is called the function's indefinite integral. Once you select a part of the curve to analyze, the area under the curve over that region is a number, and is called a definite integral. The formulas needed to form a function's indefinite integral depend on the function under which you are trying to find the area. If the function is a straight line or involves only straight line segments, you can find the area underneath it using formulas from geometry, such as the area of a trapezoid or of a triangle.

Cumulative (Distribution) Functions

Example Int-1.

Consider the function y = f(x) = 0.02x,   0<x<10 (Figure 18). What is the area under the curve?



$$F(x) = \int f(x)dx = \int .02x\,dx = 0.02 * \frac{x^2}{2} = 0.01x^2$$

The area between a=0 and b=10 is then

F(b) - F(a) = F(10) – F(0) = 0.01(100) – 0 = 1.

In statistics, probabilities for continuous random variables are defined by areas under curves, so that when the total area under f(x) equals 1, the f(x) can be used as a probability density function. In such a case, F(x) is called the cumulative distribution function (CDF) of X, and its values are interpreted as the probability that a random variable X takes on values less than or equal to x.

Review Problem Int-1.

What is the area under the curve f(x) = 0.02x between a = 1 and b = 6?

Example Int-2.

What is the area under $y = f(x) = 0.02x^2$ between $a = 0$ and $b = 10$ (Figure 19)?

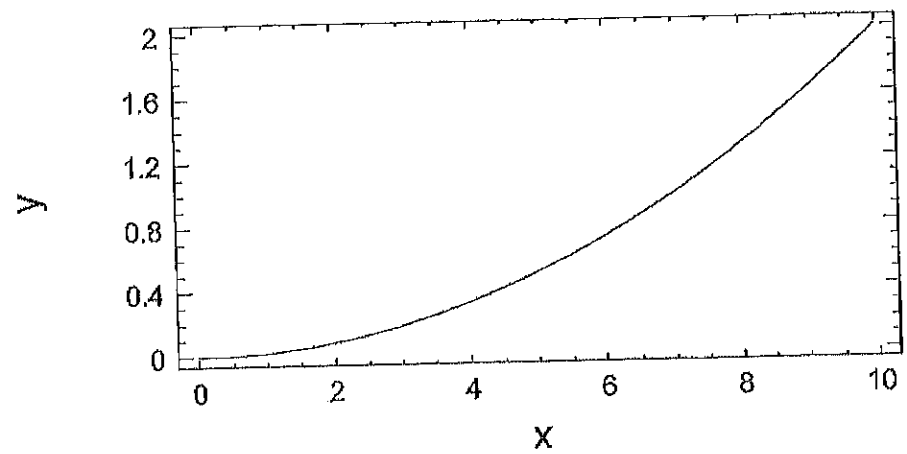


Figure 19.

When $f(x) = 0.02x^2$, the rule on page ~~43~~ 15 says that

$$F(x) = 0.02x^3/3$$

The area between a=0 and b=10 is $F(b) - F(a) = F(10) - F(0) = [0.02(1000)/3] - 0 = 20/3 = 6\frac{2}{3}$.

That area is used in statistics – it is the average value of x when the probabilities of x follow the function in Example Int-1.

Review Problem Int-2.

What is the area under the curve $y = f(x) = 0.02x^3$ between $a = 0$ and $b = 10$?

4. Sums of Series of Constants

How do you add up a list of costs or revenues? That can be a problem when the list is very large or infinite, and occurs when financial considerations must be considered over an unbounded planning horizon. The concept is closely linked with the idea of discounting future costs and returns, an idea called *present value* in finance. In this section, we deal with ways of adding up such lists of values, where the values are numbers, as opposed to being algebraic expressions involving variables. While it is certainly easy enough to add up an array of values using a calculator or Excel, when the list is known and not particularly long, it is useful to learn the general way to deal with series in order to equip you to deal with more arbitrary examples.

How you find the sum of a series of values depends on the relationship between consecutive values in the series. In this section, we consider two cases:

1. Arithmetic series
2. Geometric series
3. Harmonic Series

Example AP Series

In an arithmetic series, each item is a fixed amount larger or smaller than the previous item .

ARITHMETIC SERIES – GENERAL FORMULA

$$S_n = a + (a+d) + (a+2d) + (a+3d) + \ldots\ldots\ldots + (a + (n-1)d)$$

More formally, if we let

a denote the first term in the series

d denote the difference between successive terms, and

n denote the number of terms in the series, so that

$L = a + (n-1)d$ is the last term in the series,

Then $(a+L)/2$ is the average of the terms of the series, and S_n, the sum of the series with n terms, is

$$S_n = n(a + L)/2$$

Instead of computing the average, it is also possible to represent the sum of the series using the first term and the difference between successive terms by substituting the formula for L in the above expression, yielding

$$S_n = n[2a + d(n-1)]/2$$

Example 1 : The sum of the first 20 odd numbers;

$1 + 3 + 5 + 7 + \ldots\ldots\ldots\ldots\ldots\ldots$

Using the notation above: $a = 1$, $d = 2$, $n = 20$

The last term $L = a + (n-1)d = 1 + (20-1)*2 = 1 + 38 = 39$

The sum of the series : $S_{20} = n(a + L)/2 = 20(1 + 39)/2 = 400$

$= n[2a+d(n-1)]/2 = 20[2+2(19)]/2 = 400$

Example 2 : Sum of the first 20 even numbers ?

GEOMETRIC SERIES

In an arithmetic series, the difference between successive terms is a constant. In a geometric series, the ratio between successive terms is a constant. A geometric series describes a sequence of values subject to compound interest, either from investment returns or from discounting, and the latter is the more common application. Money has time value, because it can be used to earn interest and returns from investment activities. A dollar received a year from now is worth less than a dollar received today, because if you received the dollar today, you could earn an extra year's worth of interest or return from it. If i denotes the annual interest rate, then a dollar a year from now is only worth $1/(1+i)$ dollars to you right now, because investing one dollar today would yield $(1+i)$ dollars a year from now. In financial applications, the sum of a series of discounted future dollar values is called its *present value*.

Example Series-2

Assume that you won a lottery that offers you a series of four annual payments of $1000 each, the first one to be paid now. Also assume that because of investment alternatives, a dollar received a year from now is worth only 90 cents to you, meaning that you use an interest rate of 11.11%.

The series of four payments, S_4, is worth

$$\begin{aligned} S_4 &= \$1000 + (0.9)*\$1000 + (0.9)^2*\$1000 + (0.9)^3*\$1000 \\ &= \$1000 + \$900 + \$810 + \$729 \\ &= \$3439 \end{aligned}$$

Each term is 90% of the previous term – this is a four-term geometric series and 0.9 is the ratio of successive terms.

GEOMETRIC SERIES – GENERAL FORMULA

Because the terms in a geometric series are not equally spaced apart, the average of the series is not the average of the first and last terms. As a result, the formula for calculating the sum of a geometric series is different from the formula for calculating the sum of an arithmetic series.

$S_n = a + ar + ar^2 + ar^3 + \ldots + ar^{(n-1)}$

If we let

a denote the first term in the series,
r denote the ratio of successive terms, and
n denote the number of terms in the series, so that
$L = ar^{n-1}$ is the last term in the series,

Note that if *i* is the interest rate, then $r = 1/(1+i)$.

Then S_n, the sum of the n-term geometric series is

$$S_n = \frac{a - rL}{1 - r} = \frac{a(1 - r^n)}{1 - r}$$

Using sigma notation, the sum of an n term geometric series, beginning with A and with a term-to-term ratio of r is written as

$$S_n = \sum_{j=1}^{n} ar^{j-1}$$

For Example Series-2, the sigma-notation form of the problem, which has an initial value of \$1000, a term-to-term ratio of 0.9, and four terms, is

$$S_4 = \sum_{j=1}^{4} \$1000(0.9)^{j-1}$$

GEOMETRIC SERIES – INFINITE CASE GENERAL FORMULA

If we let

a denote the first term in the series and
r denote the ratio of successive terms, with r < 1,

Then S_∞, the sum of the infinite geometric series is

$$S_\infty = \frac{a}{1-r}$$

Example :As in Example Series-2, assume that you are to receive annual payments of $1000, with the first payment to be made today, and that dollar next year is worth only 90 cents to you today, but that the payments continue indefinitely. Then A=$1000 and r=0.9, and the present value of the stream of payments is

$$\begin{aligned} {}_\infty \quad = \quad & \$1000 \\ & + (0.9)*\$1000 \\ & + (0.9)^{2}*\$1000 \\ & + (0.9)^{3}*\$1000 \\ & + (0.9)^{4}*\$1000 \\ & + (0.9)^{5}*\$1000 \\ & + (0.9)^{6}*\$1000 \\ & + \ldots \end{aligned}$$

$$S_\infty = \frac{a}{1-r} = \frac{\$1000}{1-0.9} = \frac{\$1000}{0.1} = \$10{,}000$$

Quiz

1. a. Solve the following two equations:

$$2x + 5y = 26 \text{ — } ①$$

$$3x + 10y = 49 \text{ — } ②$$

   b. What is the slope of line (1).

2. Find the derivative of the following two functions:
   a. $f(x) = x^3 + 4x^2 + 2x + 5$
   b. $f(x) = e^{2x} + 5x^2 + 2x$

3. Find the extreme points of the following function and specify the nature of the extreme points:

$$f(x) = x^3 - 27x + 100$$

4. Find the area of $f(x)$ from 2 to 5

$$f(x) = 3x$$

5. Find the sum of the following series:
   a. $100 + (100 + 5) + \{100 + (2 * 5)\} + \cdots + \{100 + (19 * 5)\}$
   b. $100 + (0.95)100 + (0.95)^2 100 + \cdots + (0.95)^{10} 100$
   c. $(0.95)100 + (.95)^2 100 \ldots\ldots.$